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# INSTABILITY OF A STEADY-STATE REGIME FOR THE COMBUSTION OF PETROLEUM IN A POROUS MEDIUM

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A system of equations for multiphase nonisothermal filtration is used to examine the problem of dynamic instability, as well as the self-excited oscillations of a plane fuel-combustion front in a porous medium.

In a number of experiments designed to model the process of combustion within a combustion front, we observe variations in the velocity at which the petroleum-combustion front moves [1-3]. The existence of oscillatory instability exhibited by the plane front of steady combustion and the self-excited regular oscillations generated in this way are long since familiar from examples of the combustion of powders and nongaseous systems [4, 5], and from filtration combustion of metals [6]; however, unlike these last examples, no theory has yet been constructed for the instability and self-induced oscillations of a fuel-combustion front in a porous medium. Analytical results in this area have been obtained within the framework of a steady-state postulation of the problem [7, 8]; the nonsteady effects have been investigated primarily with numerical methods [9, 10].

In the general case, the theoretical investigation of the process of combustion within a combustion front must be based on a system of differential equations for multiphase nonisothermal filtration, which includes the continuity equations for petroleum as well as for a multicomponent fuel that undergoes changes as a consequence of oxidation, distillation, cracking, etc., both for water and steam, for an inert gas and an oxidizer pumped into the fuel, for gaseous reaction products, as well as an equation for the temperature and laws of filtration. The complexity of the mathematical analysis of such a system of nonlinear equations is obvious and, therefore, use is normally made of a series of simplifying assumptions (see, for example, the review in [1]).

Statement of the Problem. It is assumed in this paper that the petroleum is an incompressible liquid made up of only a single component and that it enters into the oxidation process in accordance with the reaction $H + \nu_1 O_2 \rightarrow \nu_3 \Pi$, where $\Pi$ denotes the gaseous reaction products. It is assumed that these last exhibit the same thermophysical properties as an oxidizer and an inert gas. We examine the "dry" combustion of the petroleum, achieved solely by forcing air into the interstitial space.

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Within the framework of a monotemperature model and in quasihomogeneous approximation in a coordinate system moving at the speed of the combustion front, the system of equations for the filtration combustion of petroleum in a porous medium can be written in the following form [8, 9]:

$$\frac{\partial}{\partial t}(ms\rho_g) + \Phi'(t)\frac{\partial}{\partial x}(ms\rho_g) + \frac{\partial}{\partial x}(\rho_g V_g) = \rho_o W(s,\ a,\ T),\ p = R\rho_g T m^{-1},$$

$$\frac{\partial}{\partial t}(msa\rho_g) + \Phi'(t)\frac{\partial}{\partial x}(msa\rho_g) + \frac{\partial}{\partial x}(a\rho_g V_g) = -\mu\rho_o W(s,\ a,\ T),$$

$$\frac{\partial}{\partial t}(m(1-s)\rho_o) + \Phi'(t)\frac{\partial}{\partial x}(m(1-s)\rho_o) + \frac{\partial}{\partial x}(\rho_o V_o) = -\rho_o W(s,\ a,\ T), \tag{1}$$

$$C\frac{\partial T}{\partial t} + C\Phi'(t)\frac{\partial T}{\partial x} + (\rho_g c_g V_g + \rho_o c_o V_o)\frac{\partial T}{\partial x} = \lambda\frac{\partial^2 T}{\partial x^2} + Q\rho_o W(s,\ a,\ T),$$

$$C = (1-m)\rho_r c_r + m(1-s)\rho_o c_o + ms\rho_g c_g,\ V_o = -\frac{kk_o(s)}{\mu_o'(T)}\frac{\partial p}{\partial x},$$

$$V_g = -\frac{kk_g(s)}{\mu_g'(T)}\frac{\partial p}{\partial x}.$$

In writing the equations for the temperature we assume that the specific heat capacities for the petroleum and the gas coincide. For the reaction considered in this paper the equation $c_o = c_g$ is rigorously satisfied if the thermal effect of the reaction Q is independent of T.

Analysis of system (1) is significantly simplied when the velocity of the gas is substantially greater than the velocity of the front ($V_g \gg \Phi'(t)$). In this case, the characteristic time of establishing the steady distribution of the pressure field $\tau_p$ (or of the density field) is considerably less than the relaxation time $\tau_T$ for the temperature field. Indeed, let the spatial scale of the problem be $L \sim \kappa/U$, where $\kappa$ is the thermal conductivity of the medium and U is the steady velocity of the combustion front. It is obvious that $\tau_T \sim L^2/\kappa$, for $\tau_T \sim L/U$, while $\tau_p \sim L^2/K$, where K is the coefficient of piezoconductivity, or $\tau_p \sim L/V_g$, so that consequently the inequality $\tau_p \ll \tau_T$ corresponds to the inequality $U \ll V_g$. Assuming this latter condition to be satisfied, we further assume that the relative deviations in the rate of petroleum filtration and saturation from steady values, resulting from the fluctuations in temperature, are considerably smaller than the relative perturbations in the velocity of the combustion front. This last condition is always valid in the kinetic regime, when the relationship between the rate of the reaction and the temperature is considerably stronger than an analogous relationship for the viscosity of the petroleum. Thus, we will assume that $\rho_g$ and s are explicitly independent of time. It is obvious that in this case the problem of analyzing system (1) is significantly simplified, since it is necessary to consider only the nonsteady nature of the temperature field, regarding the filtration motion in this case as steady.

Within the scope of the assumptions made so far, the first three equations of system (1) can be rewritten in simpler form by

$$\frac{\partial}{\partial x}(\rho_g V_g) = \rho_o W,\ \frac{\partial}{\partial x}(a\rho_g V_g) = -\mu\rho_o W,\ \frac{\partial}{\partial x}(\Phi'(t)m(1-s) + V_o) = -W. \tag{2}$$

Limiting ourselves to the accompanying forced filtration, where the gas flow $G_*$ behind the combustion front is given, as is the oxidizer concentration $a_*$, we can write the first integrals of system (2):

$$\rho_g V_g + \rho_o(\Phi'(t)m(1-s) + V_o) = -G_*,\ a\rho_g V_g + \mu(\rho_g V_g + G_*) = -a_* G_*. \tag{3}$$

Using the first integral in (3) and the inequality $\rho_g \ll \rho_o$, let us transform the equation of heat balance in system (1):

$$[(1-m)\rho_r c_r + m(1-s)\rho_o c_o]\frac{\partial T}{\partial t} + [(1-m)\rho_r c_r\Phi'(t) - c_o G_*]\frac{\partial T}{\partial x} = \lambda\frac{\partial^2 T}{\partial x^2} + Q\rho_o W. \tag{4}$$

In the assumption of a strong relationship between the rate of chemical reaction and the temperature, we approximate W with the δ-function

$$W(s,\ a,\ T)=A(T)\,\delta(x),\ A(T)\sim\exp(-E/2RT), \tag{5}$$

where A(T) is determined from an analysis of the structure of the combustion zone.

Thus, in this problem the zone of chemical reaction is treated as a discontinuity surface at which the temperature is continuous, while the saturation of the gas and of the petroleum flow experiences discontinuity, as do the gas and the oxidizer, and also the heat flow. Limiting ourselves to an examination of the kinetic combustion regime [6], we assume that we have complete combustion of the petroleum behind the front (x > 0, s = 1), in which case we have the oxidizer on both sides of the front. The second term in (3) imposes a limitation on the condition for the realization of such a regime in the form

$$a_2G_*>\mu\rho_0(\Phi'(t)\,m(1-s)+V_0)\ (a_2\equiv a_*,\ a_1>0). \tag{6}$$

Here and below, the subscript 1 denotes the quantities where x < 0, while the subscript 2 is used for x > 0.

Having integrated (4) and the last equation in (2), with consideration of the presentation in (5), as well as of the condition of continuity for the temperature field, we derive the expressions for the jump in the heat flow in the reaction zone where x = 0, and for the mass rate of petroleum combustion we will have

$$\lambda\frac{\partial T_1}{\partial x}-\lambda\frac{\partial T_2}{\partial x}=Q\rho_0A(T(0,\ t)), \tag{7}$$

$$\rho_0(\Phi'(t)\,m(1-s_1)+V_0)=\rho_0A(T(0,\ t)). \tag{8}$$

To close Eq. (4) with a source in the form of (5), we have to find an expression for the nonsteady velocity Φ'(t) of combustion-front motion as a function of temperature. For this we will use relationship (8) and the inequalities

$$\frac{dA}{dT}\gg\frac{dV_0}{dT}\sim\Phi'(t)\frac{ds_1}{dT}\quad\left(\frac{E}{RT}\gg\frac{T}{\mu_0'}\,\frac{d\mu_0'}{dT}\right), \tag{9}$$

where E is the activation energy for chemical reaction. Relationship (8) under steady-state conditions is written in the form

$$\rho_0(Um(1-s_1^0)+V^0)=\rho_0A(T_{\text{f}}). \tag{10}$$

Having found the difference between (8) and (10), and having given the function A(T) in the form

$$A(T)=A(T_{\text{f}})\exp\left[\frac{E}{2RT_{\text{f}}}(T-T_{\text{f}})\right]$$

with consideration of inequalities (9), we find an expression for the velocity Φ'(t):

$$\Phi'(t)=U+\frac{A(T_{\text{f}})}{m(1-s_1^0(0))}\left(\exp\left[\frac{E}{2RT_{\text{f}}}(T-T_{\text{f}})\right]-1\right). \tag{11}$$

After substitution of (11) into Eq. (4), we can reduce the examination of the nonsteady filtration of petroleum combustion in a porous medium to an analysis of the two equations for the temperatures $T_1$ ahead of the front (x < 0) and $T_2$ behind the front (x > 0), as well as for the condition x = 0 at the discontinuity, and for infinity:

$$C_i\frac{\partial T_i}{\partial t}+[(1-m)\,\rho_rc_rU+c_gG_*]\frac{\partial T_i}{\partial x}+\frac{(1-m)\,\rho_rc_rA(T_{\text{f}})}{m(1-s_1^0(0))}\times$$
$$\times\left(\exp\left[\frac{E}{2RT_{\text{f}}}(T(0,\ t)-T_{\text{f}})\right]-1\right)\frac{\partial T_i}{\partial x}=\lambda\frac{\partial^2T_i}{\partial x^2}; \tag{12}$$
$$i=1,\ 2;\ C_1=(1-m)\,\rho_rc_r+m(1-s_1^0)\,\rho_0c_0=\text{const},\ C_2=(1-m)\,\rho_rc_r,$$
$$T_1(0,\ t)=T_2(0,\ t),\ T_1(-\infty,\ t)=T_{10},\ T_2(\infty,\ t)<\infty,$$

$$\lambda \frac{\partial T_1}{\partial x}(0, t) - \lambda \frac{\partial T_2}{\partial x}(0, t) = Q\rho_0 A(T_f) \exp\left[\frac{E}{2RT_f}(T(0, t) - T_f)\right]. \tag{12}$$

Using the following quantities as dimensionless variables and parameters

$$\theta_i = \frac{T_i - T_{10}}{T_f - T_{10}}, \quad \tau = \frac{t(1-m)\rho_r c_r A^2(T_f)}{\lambda m^2 (1 - s_1^0(0))}, \quad z = \frac{x(1-m)\rho_r c_r A(T_f)}{\lambda(1 - s_1^0(0))m},$$

$$\alpha = \frac{E}{2RT_f}\left(1 - \frac{T_{01}}{T_f}\right), \quad \delta = \frac{m(1 - s_1^0)\rho_0 c_0}{(1-m)\rho_r c_r}, \tag{13}$$

$$\beta = \frac{m(1 - s_1^0(0))[(1-m)\rho_r c_r U - c_g G_*]}{(1-m)\rho_r c_r A(T_f)},$$

we can write system (12) in the following form:

$$(1+\delta)\frac{\partial\theta_1}{\partial\tau} + \beta\frac{\partial\theta_1}{\partial z} + [e^{\alpha(\theta_1(0,\tau)-1)} - 1]\frac{\partial\theta_1}{\partial z} = \frac{\partial^2\theta_1}{\partial z^2} \quad (-\infty < z < 0),$$

$$\frac{\partial\theta_2}{\partial\tau} + \beta\frac{\partial\theta_2}{dz} + [e^{\alpha(\theta_1(0,\tau)-1)} - 1]\frac{\partial\theta_2}{\partial z} = \frac{\partial^2\theta_2}{\partial z^2} \quad (0 < z < \infty), \tag{14}$$

$$\theta_1(0, \tau) = \theta_2(0, \tau), \quad \frac{\partial\theta_1}{\partial z}(0, \tau) - \frac{\partial\theta_2}{\partial z}(0, \tau) = \beta e^{\alpha(\theta_1(0,\tau)-1)},$$

$$\theta_1(-\infty, \tau) = 0, \quad \theta_2(\infty, \tau) < \infty.$$

In writing the boundary condition for the temperature-gradient jump we employ the equation which links the temperature $T_f$ at the front to the temperature $T_{10}$ at an infinite distance from the front:

$$T_f = T_{10} + Q\rho_0 A(T_f)[(1-m)\rho_r c_r U - c_g G_*]^{-1}.$$

The steady-state solution of problem (14) is given by

$$\theta_1^0(z) = e^{\beta z}, \quad \theta_2^0(z) = 1. \tag{15}$$

Linear Analysis of Stability. The nonsteady solutions of system (14) will be sought in the form of the sum of the steady-state solution and the small perturbations:

$$\theta_1(z, \tau) = \theta_1^0(z) + u_1(z, \tau), \quad \theta_2(z, \tau) = \theta_2^0(z) + u_2(z, \tau). \tag{16}$$

After having substituted (16) into (14), and with subsequent linearization for the small quantities $u_1$ and $u_2$, we obtain the equations

$$(1+\delta)\frac{\partial u_1}{\partial\tau} + \beta\frac{\partial u_1}{\partial z} + \alpha u_1(0, \tau)\frac{\partial\theta_1^0}{\partial z} = \frac{\partial^2 u_1}{xz^2}, \quad \frac{\partial u_2}{\partial\tau} + \beta\frac{\partial u_2}{\partial z} = \frac{\partial^2 u_2}{\partial z^2} \tag{17}$$

and the boundary conditions

$$u_1(0, \tau) = u_2(0, \tau), \quad \frac{\partial u_1}{\partial z}(0, \tau) - \frac{\partial u_2}{\partial z}(0, \tau) = \alpha\beta u_1(0, \tau), \tag{18}$$

$$u_1(-\infty, \tau) = 0, \quad u_2(\infty, \tau) < \infty.$$

The solution of the linear problem (17)-(18) can be written in the form

$$u_1 = Ae^{i\omega\tau}\left[(1 - i\alpha\beta\omega^{-1}(1+\delta)^{-1})\exp\left[(\beta + \sqrt{\beta^2 + i4(1+\delta)\omega})\frac{z}{2}\right] + i\alpha\beta\omega^{-1}(1+\delta)^{-1}\exp\beta z\right], \tag{19}$$

$$u_2 = Ae^{i\omega\tau}\exp\left[(\beta - \sqrt{\beta^2 + i4\omega})\frac{z}{2}\right],$$

where $\Omega = i\omega$ satisfies the dispersion relationship

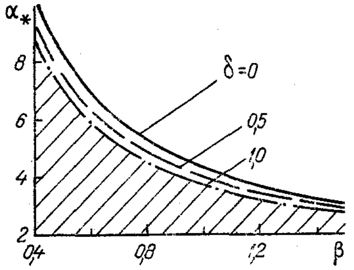

Fig. 1

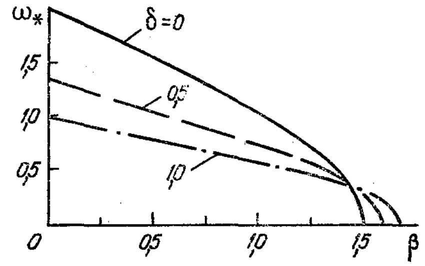

Fig. 2

Fig. 1. Curves showing the neutral stability ($\mathrm{Re}\,\Omega = 0$) in the parametric plane $\alpha_*$, $\beta$ for various values of the parameter $\delta$.

Fig. 2. Dimensionless frequency $\omega_* = \mathrm{Im}\,\Omega$ ($\mathrm{Re}\,\Omega = 0$) as a function of $\beta$ for various values of the parameter $\delta$.

$$(1+\delta)\Omega\left(\sqrt{\beta^2+4(1+\delta)\Omega}+\sqrt{\beta^2+4\Omega}-2\alpha\beta\right)+\alpha\beta\left(\sqrt{\beta^2+4(1+\delta)\Omega}-\beta\right)=0. \qquad (20)$$

We see from solution (19) that the steady-state solution (15) is stable if $\mathrm{Re}\,\Omega < 0$. In Fig. 1, for the parametric region $\alpha_*$, $\beta$ we see the curves of neutral stability ($\mathrm{Re}\,\Omega = 0$), obtained through numerical analysis of the dispersion relationship (20). These curves correspond to various values of the parameter $\delta$, which characterizes the difference in heat capacity of the medium in front of and behind the combustion front. The dashed line shows the region of stability. We see that as the parameter $\delta$ increases, the region of instability expands. It becomes clear from Fig. 2 that with a reduction in the parameter $\beta$ the frequency $\omega_*$ increases ($\omega_* = \mathrm{Im}\,\Omega$).

When the heat capacities of the medium on the two sides of the front coincide ($\delta = 0$), the dispersion relationship assumes the simple form

$$4\Omega^2+(\beta+4\alpha-\alpha^2\beta)\beta\Omega+\alpha\beta^2(\beta+\alpha-\alpha\beta)=0. \qquad (21)$$

Equation (21) has two roots

$$\Omega_{1,2}=-\frac{\beta(\beta+4\alpha-\alpha^2\beta)}{8}\pm\frac{\beta}{8}\sqrt{(\beta+4\alpha-\alpha^2\beta)^2-16\alpha(\beta+\alpha-\alpha\beta)}. \qquad (22)$$

Let the expression in (22) under the radical sign be less than zero, in which case the instability condition ($\mathrm{Re}\,\Omega > 0$) is written in the form $\beta + 4\alpha - \alpha^2\beta > 0$, while the neutral-stability curve $\beta = 4\alpha(\alpha^2 - 1)^{-1}$ corresponds to the values of $\alpha_* = 2/\beta + \sqrt{1 + 4/\beta}$. When $\beta = 1$, we obtain the results $\alpha_* = 2 + \sqrt{5}$, corresponding to the combustion of nongaseous systems [5].

The region of parameters for which $\beta > 4\alpha(\alpha^2 - 1)^{-1}$ and $\beta < 8\alpha(1 + \alpha)^{-2}$ corresponds to the oscillatory nature of stability loss. We see from relationship (22) that on the neutral-stability curve the oscillation frequency is defined in the following manner: $\omega_* = (\beta/2)\cdot\sqrt{\alpha_*(\beta + \alpha_* - \alpha_*\beta)}$. By means of (13) we can determine the period of temperature oscillation:

$$T=\frac{2\pi\lambda m^2(1-s_1^0(0))}{\omega_*(1-m)\rho_r c_r(m(1-s_1^0(0))U-|V^0|)}. \qquad (23)$$

Calculations with formula (23) yield good qualitative agreement with experimental results obtained on laboratory models simulating the actual conditions [3]. Thus, for example, with $U = 0.36$ m/h, $|V^0| = 0.01U$, $\lambda/\rho_r c_r = 10^{-3}$ m²/h, $\alpha_* = 5$, $\beta = 0.83$, $\omega_* = 1.2$, $m = 0.8$, $s_1{}^0 = 0.72$, we obtain the oscillation period $T \approx 40$ min [3].

In order to answer the question of how the gas filtration affects the stability of the process, with consideration of (9) let us transform the expression for the parameter $\beta$ in (13) as follows:

$$\beta=\frac{1-c_g G_*[(1-m)\rho_r c_r U]^{-1}}{1-|V^0|[m(1-s_1^0)U]^{-1}}. \qquad (24)$$

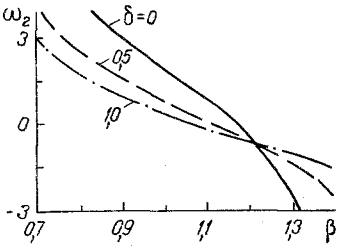

Fig. 3

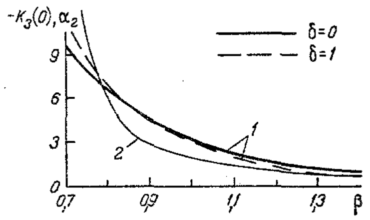

Fig. 4

Fig. 3. Parameter $\omega_2$ as a function of $\beta$ for various values of the parameter $\delta$.

Fig. 4. Parameter $\alpha_2$ as a function of $\beta$ (curves 1), and the coefficient $-K_3(0)$ as a function of $\beta$ with a rightward shift of 0.6 along the abscissa (curves 2).

As we can see from Fig. 1, an increase in the parameter $\beta$ may lead to the destabilization of the steady-state combustion regime. It is clear from (24) that despite the fact that the gas and petroleum flows were moving in the same direction, the motion of the petroleum, unlike that of the accompanying gas flow, is a destabilizing factor.

Self-Induced Oscillations. It is obvious that the experimentally observed oscillations in the velocity of the petroleum combustion-front motion are a manifestation of a self-excited oscillatory regime whose characteristics can be determined through an analysis of the nonlinear system of equations and boundary conditions found in (14). In the general case, such a study can be carried out only with numerical methods; however, near the instability threshold, when $0 < \alpha - \alpha_* \ll 1$, the methods from the theory of perturbations [11, 12] make it possible analytically to construct a periodic solution of Eqs. (14). Let us note that the successful study of the self-excited oscillatory and spin combustion regimes for nongaseous systems and of systems with diffusion by the methods of perturbations was performed in [13-16].

We will present the solution of system (14) in the form of the sums

$$\theta_1(z, \tau) = \exp(\beta z) + v_1(z, \tau), \quad \theta_2(z, \tau) = 1 + v_2(z, \tau). \tag{25}$$

We should expect that near the threshold of self-excited oscillations ($\alpha = \alpha_*$, $\omega = \omega_*$), the terms $v_1(z, \tau)$ and $v_2(z, \tau)$ differ little from the solutions for the linear equations (17)-(18). The frequency of the self-excited oscillations will also differ little from the quantity $\omega_*$ derived within the framework of the linearized problem at the boundary of neutral stability. Having introduced the frequency $\omega_*$ for the self-excited oscillations into Eq. (14) by means of the transformation $\tau' = \omega\tau$ and having substituted sums (25) into (14), we obtain the following nonlinear equations for $v_1$ and $v_2$:

$$\omega \frac{\partial v_i}{\partial \tau'} + \beta \frac{\partial v_i}{\partial z} + [\exp(\alpha v_1(0, \tau')) - 1] \left( \frac{\partial v_i}{\partial z} + (2 - i) \beta \exp(\beta z) \right) = \frac{\partial^2 v_i}{\partial z^2}; \quad i = 1, 2. \tag{26}$$

The boundary conditions have the form

$$\begin{gathered} v_1(-\infty, \tau') = 0, \quad v_2(\infty, \tau') \leqslant \infty, \quad v_1(0, \tau') = v_2(0, \tau'), \\ \frac{\partial v_1}{\partial z}(0, \tau') - \frac{\partial v_2}{\partial z}(0, \tau') = \beta (\exp(\alpha v_1(0, \tau')) - 1). \end{gathered} \tag{27}$$

The solution of system (26)-(27) periodically with respect to time, frequency $\omega$ of the self-excited oscillations, and the bifurcated parameter $\alpha$ will be sought in the form of an exponential expansion of the small parameter $\varepsilon$ [13-16]. Dropping all of the calculations, which should form the subject matter of a separate paper, we will cite only the final results:

$$\begin{gathered} v_i(z, \tau', \varepsilon) = \frac{\varepsilon}{2A} [u_i(z, \tau'/\omega, \omega_*, \alpha_*) + \overset{*}{u_i} (z, \tau'/\omega, \omega_*, \alpha_*)] + \\ + \varepsilon^2 [K_{3i-2}(z) \cos 2\tau' + K_{3i-1}(z) \sin 2\tau' + K_{3i}(z)] + O(\varepsilon^2); \\ i = 1, 2; \quad \omega = \omega_* + \omega_2 \varepsilon^2 + O(\varepsilon^2), \quad \alpha = \alpha_* + \alpha_2 \varepsilon^2 + O(\varepsilon^2). \end{gathered} \tag{28}$$

The functions $u_1$ and $u_2$ have been determined from (19). The specific form of the function $K_1(z)$-$K_6(z)$ is omitted here.

We see from Fig. 3 that with a reduction in $\beta$ the nonlinear correction for the frequency is increased. The value of $\alpha_2$ is always positive, which indicates soft self-excitation of the oscillations.

The chief significance of the theory is the answer to the question of how the effects of nonlinearity and filtration impinge on the mean velocity of the nonstationary combustion front. According to (11), the velocity $\Phi'(t)$ is determined from the formula

$$\Phi'(\tau') = U + \frac{A(T_\Phi)}{m(1 - s_1^0)} [\exp(\alpha v_1(0,\ \tau',\ \varepsilon)) - 1].$$

Hence it follows that the average correction for U can be determined in the following manner: $\langle\Phi'(\tau') - U\rangle = \alpha_* \varepsilon^2 K_3(0) A(T_f)\ [m(1 - s_1{}^0)]^{-1}$. As we can see from Fig. 4, the mean velocity in a nonstationary regime diminishes, which is in agreement with the results of [13], wherein for small $\beta$ this reduction may be rather great.

## NOTATION

$\rho$, density; V, velocity; m, porosity; s, saturation; $a$, oxidizer concentration; p, pressure; T, temperature; C, heat capacity of a volume unit; c, heat capacity of a unit of mass; $\mu$ and $\nu$, stoichiometric coefficients; W, rate of the chemical reaction; Q, heat of the reaction; $\lambda$, coefficient of thermal conductivity; R, gas constant; k, coefficient of permeability; $\mu'$, viscosity; x, a spatial coordinate; z, dimensionless coordinate in (13); t, time; $\tau$, relaxation time and dimensionless time; U and $\Phi'(t)$, steady and nonsteady combustion fronts; $G_*$, gas flow at infinity behind the front; $a_*$, oxidizer concentration at infinity behind the front; L, characteristic scale; A(T), function in (5); E, activation energy; $\alpha$, $\delta$, and $\beta$, dimensionless parameters in (13); $\theta$, dimensionless temperature in (14); $\omega$ and $\Omega$, dimensionless frequencies; $\omega_2$, $\alpha_2$, and $\varepsilon$, parameters in (28). The subscripts are as follows: g, gas; o, oil (petroleum); r, solid skeleton; *, parameters on the curve for neutral stability and in (3); $\Phi$, the temperature of the front, under steady conditions; 10, temperature at an infinite distance ahead of the front; the superscripts are as follows: 0, steady-state value and *, complex conjugate.

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# ANALOGY BETWEEN TWO MATHEMATICAL MODELS IN THE THEORY OF TWO-PHASE FILTRATION

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It is demonstrated that the general model of nonequilibrium two-phase filtration [1] describes the filtration of a two-phase fluid in the simplest model of a fissured and porous medium.

1. In mathematical models a fissured-porous material is treated as a medium exhibiting dual porosity with each type, i.e., fissured and porous [2, 3], present in each "elementary" macrovolume of the medium. Each of these two distinct types of media (assuming that the other is replaced with a solid skeleton) exhibits a unique porosity and permeability. A fissured-porous medium is the limiting case in which the porosity of the fissures and the permeability of the block tend toward zero.

The combined filtration of two nonmixing incompressible liquids (water and petroleum) in such a medium are described by a system of continuity equations for each phase in the fissures and in the porous blocks as well as in the generalized Darcy's laws for rates of phase filtration in the fissures:

$$M\partial_t s + \nabla \mathbf{u}_1 + q = 0, \quad M\partial_t (1 - s) + \nabla \mathbf{u}_2 - q = 0,$$
$$m\partial_t \sigma - q = 0, \quad \mathbf{u}_i = -\frac{k}{\mu_i} f_i(s) \nabla p \;\; (i = 1,\ 2). \tag{1}$$

Here s and σ denote, respectively, the saturation with water of the fissures and the blocks; q is the volumetric density of the water flow from the fissures to the blocks (i.e., the volume of the water overflowing into a unit volume of the medium per unit time). The direct transfer of liquid to the blocks is not taken into consideration. The fourth continuity equation (for the oil in the blocks) has actually already been accounted for in that the petroleum flow from the blocks to the fissures is assumed to be equal to the water flowing in the opposite direction.

The closing relationship for system (1) must determine the magnitude of the return flows q between the media, which, generally speaking, is a function of the history of the process [i.e., s(t) and σ(t)] at this point. Various simplifying assumptions relative to q were employed in [3-5] which made it possible to write out the closing relationship. It is proposed in [4] to treat q as a universal function (for the given properties of the medium) of the time that the block spends behind the front formed by the water. It was assumed in [5] that q is defined by the instantaneous saturations of the fissures and the blocks at a given point: $q = q(s, \sigma)$. In the following we will deal only with this latter model.

2. In a state of equilibrium between the blocks and the fissures we have $q(s, \sigma) = 0$ and $\sigma = \varphi(s)$ (the function $\varphi$ is found from the condition of equality for the capillary pressures in the media). For minor deviations from equilibrium, we assume linearization (see Fig. 1):

$$q(s_0,\ \sigma) \simeq q(s_0,\ \sigma_0) + \partial_\sigma q|_{\sigma_0} (\sigma - \sigma_0) = -\partial_\sigma q|_{\varphi(s_0)} (\varphi(s_0) - \sigma).$$

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